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Long-distance effects in $B \to V\gamma$ radiative weak decays

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Abstract

A systematic approach to long-distance effects in exclusive radiative weak B decays is presented, based on a combination of the heavy quark limit with perturbative QCD. The dominant long-distance effects, connected with weak annihilation and W-exchange topologies, can be computed in a model-independent way using experimental data on B radiative leptonic decays. Nonfactorizable corrections vanish in the chiral limit and to leading twist. The left-handed photon amplitudes are shown to be enhanced relative to the right-handed ones, both in the long- and short-distance parts of the $\bar{B}$ decay amplitudes. Recent CLEO data on $B \to K^*\gamma$ decays are consistent with Standard Model estimates of the long-distance contributions, and disfavor an enhanced gluonic penguin contribution. We discuss the implications of our results for the extraction of $|V_{td}|$.

I. INTRODUCTION

Rare radiative decays of B mesons received considerable theoretical attention due to their special sensitivity to physics beyond the Standard Model. Alternatively, if the validity of the SM is taken for granted, these decays can offer useful information on the magnitude of the CKM parameters. So far, most of the theoretical effort has been concentrated on aspects of inclusive decays $b \to s\gamma$, which can be treated with the help of an operator product expansion (OPE) combined with the heavy quark expansion [1]. The corresponding strong interaction effects can be quantified in terms of a few nonperturbative matrix elements. Experimental results on the branching ratio for inclusive $B \to X_s\gamma$ decays [2,3] appear to be in agreement with the predictions of the Standard Model.

On the other hand, the treatment of exclusive radiative decays such as $B \to K^*\gamma$ or $B \to \rho\gamma$ is considerably more difficult. This is due to the fact that bound state effects must be taken into account, which are essentially nonperturbative in nature. In addition to the matrix element of the penguin operator (short-distance component), a nonlocal contribution must be included too, arising from a product of the electromagnetic coupling with the weak nonleptonic Hamiltonian (long-distance component). These effects have been estimated using various methods, such as perturbative QCD combined with the quark model [5,6,9–11], dispersion methods [12], vector meson dominance [13,14] and QCD sum rules [15–17].

Recently the CLEO collaboration measured the branching ratios for several exclusive radiative decays $B \to V\gamma$, with $V = K^{*0}, K^{*+}$ [4], and gave upper limits on the $V = \rho, \omega$ modes. A more detailed study of these decays is therefore well motivated.

In this paper we propose a different approach to the calculation of the dominant long distance contributions to $B \to V\gamma$ decays (with V a member of the SU(3) octet of vector mesons), arising from weak annihilation and W-exchange topologies. We start by parametrizing all these decays in terms of a few common amplitudes (for each photon helicity $\lambda = L, R$), using SU(3) symmetry. It can be shown that the structure of these universal amplitudes simplifies very much to the leading order of a twist expansion, in powers of Λ/E_γ and $\Lambda/p_+ = \Lambda/m_B$, with p_+ the light-cone component of the vector meson momentum. Since the photon energy in $B \to \rho(K^*)\gamma$ decays is $E_\gamma = 2.6$ GeV, the higher twist corrections can be expected to be well under control. We find that, to leading twist, the photons emitted in $\bar{B}$ decay are predominantly left-handed. Second, the nonfactorizable corrections turn out to be computable in terms of the light-cone wavefunctions of the B and V mesons. Furthermore, in the chiral limit, the nonfactorizable corrections vanish exactly to one-loop order. The remaining factorizable contribution can be determined in a model-independent way using experimental data on radiative leptonic B decays $B \to \gamma e\nu$.

These results can be used to assess the feasibility of the determination of the ratio $|V_{td}/V_{ts}|$ from a comparison of the $B \to K^*\gamma$ and $B \to \rho\gamma$ decays. The Appendix describes certain constraints on radiative decay form-factors following from a Ward identity expressing the conservation of the electromagnetic current.

II. FLAVOR SU(3) PREDICTIONS FOR $\bar{B} \to V\gamma$ DECAY AMPLITUDES

In the Standard Model, the radiative $\bar{B} \to V\gamma$ decays (with V a member of the SU(3) octet of vector mesons) are dominated by the operators ($q = s, d$) (see, e.g., [19])

$$\mathcal{H}_{rad} = \frac{-4G_F}{\sqrt{2}} \lambda_t^{(s)} \sum_{i=7,8} C_i(\mu) \mathcal{O}_i^{(s)}(\mu) + (s \to d)\,, \tag{1}$$

$$\mathcal{O}_7^{(q)} = \frac{e}{16\pi^2} F^{\mu\nu} \bar{q} \sigma_{\mu\nu} (m_b P_R + m_q P_L) b\,, \tag{2}$$

$$\mathcal{O}_8^{(q)} = \frac{g_s}{16\pi^2} \bar{q} \sigma_{\mu\nu} (m_b P_R + m_q P_L) G^{\mu\nu} b\,, \quad P_{R,L} = \frac{1}{2}(1 \pm \gamma_5) \tag{3}$$

We denote here and in the following the combination of CKM matrix elements $\lambda_q^{(q')} = V_{qb} V_{qq'}^*$. In addition to this, there are also contributions mediated by the usual weak nonleptonic Hamiltonian

$$\begin{aligned}\mathcal{H}_W = \frac{4G_F}{\sqrt{2}} \Big\{ & \lambda_u^{(s)} [C_1 (\bar{u}b)_{V-A} (\bar{s}u)_{V-A} + C_2 (\bar{s}b)_{V-A} (\bar{u}u)_{V-A}] \\ & + \lambda_c^{(s)} [C_1 (\bar{c}b)_{V-A} (\bar{s}c)_{V-A} + C_2 (\bar{s}b)_{V-A} (\bar{c}c)_{V-A}] - \lambda_t^{(s)} \sum_{i=3}^{10} c_i Q_i^{(s)} + (s \to d) \Big\}\end{aligned} \tag{4}$$

with $(\bar{q}_1 q_2)_{V-A} (\bar{q}_3 q_4)_{V-A} = [\bar{q}_1 \gamma_\mu P_L q_2][\bar{q}_3 \gamma^\mu P_L q_4]$. In these diagrams, the photon attaches to the internal quark lines with the usual electromagnetic coupling. We will neglect in the following the contributions of the penguin operators Q_{3-10}, which are suppressed relative to those of the current-current operators by their smaller Wilson coefficients.

In a quark diagram language, all $B \to V\gamma_\lambda$ decay amplitudes are parametrized by nine independent $SU(3)$ amplitudes, for each individual photon helicity $\lambda = L, R$. They appear together in fewer combinations, such that a certain predictive power is still retained. They include penguin-type amplitudes $P_{u,c,t}$ – corresponding to topologies with a u-, c- and t-quark running in the loop, the weak annihilation (WA)-type amplitude A, the W-exchange amplitude E, penguin-annihilation amplitudes PA, and amplitudes with one insertion of the gluonic penguin operator $\mathcal{O}_8$ which will be denoted M_i (see Fig. 1). In the penguin $P_{u,c}^{(i)}$ and $M^{(i)}$ amplitudes we distinguish between diagrams with the photon coupling to the loop quark or the other emerging light quark ($i = 1$), and diagrams with the photon coupling to the spectator quark ($i = 2$). We adopt the usual phase conventions for the vector states $(\rho^+, \rho^0, \rho^-) = (u\bar{d}, \frac{1}{\sqrt{2}}(d\bar{d} - u\bar{u}), -d\bar{u})$, $(\bar{K}^{*0}, K^{*-}) = (s\bar{d}, -s\bar{u})$, $(K^{*+}, K^{*0}) = (u\bar{s}, d\bar{s})$, $\phi^{(8)} = \frac{1}{\sqrt{6}}(2s\bar{s} - u\bar{u} - d\bar{d})$, and for the heavy mesons $(B^+, B^0, B_s) = (\bar{b}u, \bar{b}d, \bar{b}s)$, $(B^-, \bar{B}^0, \bar{B}_s) = (b\bar{u}, -b\bar{d}, b\bar{s})$.

The weak annihilation amplitudes $A^{(i)}$ contribute only to the $B^\pm$ radiative decays, where they appear always in the same combination $A \equiv \frac{2}{3}A^{(1)} + \frac{2}{3}A^{(2)} - \frac{1}{3}A^{(3)}$ (see Fig. 1(a)). Writing explicitly the contributions of the penguin amplitudes one has

$$\begin{aligned} A(B^- \to \rho^- \gamma_\lambda) = & \lambda_u^{(d)} (P_{u\lambda}^{(1)} + Q_u P_{u\lambda}^{(2)} + A_\lambda) + \lambda_c^{(d)} (P_{c\lambda}^{(1)} + Q_u P_{c\lambda}^{(2)}) \\ & + \lambda_t^{(d)} (P_{t\lambda} + M_\lambda^{(1)} + Q_u M_\lambda^{(2)}) \end{aligned} \tag{5}$$

$$\begin{aligned} A(B^- \to K^{*-} \gamma_\lambda) = & \lambda_u^{(s)} (P_{u\lambda}^{(1)} + Q_u P_{u\lambda}^{(2)} + A_\lambda) + \lambda_c^{(s)} (P_{c\lambda}^{(1)} + Q_u P_{c\lambda}^{(2)}) \\ & + \lambda_t^{(s)} (P_{t\lambda} + M_\lambda^{(1)} + Q_u M_\lambda^{(2)})\,. \end{aligned} \tag{6}$$

The penguin amplitude with an internal t-quark $P_{t\lambda}$ (arising from the penguin operator $\mathcal{O}_7$) is usually called the short-distance contribution, in contrast to the long-distance amplitude, arising from nonlocal insertions of the weak Hamiltonian (4) with one photon

attachment. In a hadronic language, the penguin amplitudes $P_u^{(i)}$ receive contributions from rescattering effects of the form $B^\pm \to \{\rho^\pm\rho^0\} \to \rho^\pm\gamma$, and the charmed penguins $P_c^{(i)}$ arise from rescattering processes $B^\pm \to \{D^{*\pm}\bar{D}^{*0}\}, \{\psi\rho^0\} \to \rho^\pm\gamma$.

The $\bar{B}^0$ decay amplitudes contain the W-exchange topology, parametrized by the graphical amplitudes $E^{(i)}$ (Fig. 1(b)). All the decays discussed below contain these amplitudes in the same combination $E \equiv \frac{2}{3}E^{(2)} + \frac{2}{3}E^{(3)} - \frac{1}{3}E^{(1)}$

$$A(\bar{B}^0 \to \rho^0\gamma_\lambda) = \frac{1}{\sqrt{2}}\lambda_u^{(d)}(P_{u\lambda}^{(1)} + Q_d P_{u\lambda}^{(2)} - E_\lambda - (Q_u - Q_d)PA_{u\lambda}) \tag{7}$$

$$+\frac{1}{\sqrt{2}}\lambda_c^{(d)}(P_{c\lambda}^{(1)} + Q_d P_{c\lambda}^{(2)} - (Q_u - Q_d)PA_{c\lambda}) + \frac{1}{\sqrt{2}}\lambda_t^{(d)}(P_{t\lambda} + M_\lambda^{(1)} + Q_d M_\lambda^{(2)})$$

$$A(\bar{B}^0 \to \bar{K}^{*0}\gamma_\lambda) = \lambda_u^{(s)}(P_{u\lambda}^{(1)} + Q_d P_{u\lambda}^{(2)}) + \lambda_c^{(s)}(P_{c\lambda}^{(1)} + Q_d P_{c\lambda}^{(2)}) + \lambda_t^{(s)}(P_{t\lambda} + M_\lambda^{(1)} + Q_d M_\lambda^{(2)}) \tag{8}$$

$$A(\bar{B}^0 \to \phi^{(8)}\gamma_\lambda) = -\frac{1}{\sqrt{6}}\lambda_u^{(d)}(P_{u\lambda}^{(1)} + Q_d P_{u\lambda}^{(2)} + E_\lambda + (Q_u + Q_d - 2Q_s)PA_{u\lambda}) \tag{9}$$

$$-\frac{1}{\sqrt{6}}\lambda_c^{(d)}(P_{c\lambda}^{(1)} + Q_d P_{c\lambda}^{(2)} + (Q_u + Q_d - 2Q_s)PA_{c\lambda}) - \frac{1}{\sqrt{6}}\lambda_t^{(d)}(P_{t\lambda} + M_\lambda^{(1)} + Q_d M_\lambda^{(2)}) .$$

Finally, the corresponding $\bar{B}_s$ decays are written in terms of the amplitudes previously introduced as

$$A(\bar{B}_s \to K^{*0}\gamma_\lambda) = -\lambda_u^{(d)}(P_{u\lambda}^{(1)} + Q_s P_{u\lambda}^{(2)}) - \lambda_c^{(d)}(P_{c\lambda}^{(1)} + Q_s P_{c\lambda}^{(2)}) \tag{10}$$

$$-\lambda_t^{(d)}(P_{t\lambda} + M_\lambda^{(1)} + Q_s M_\lambda^{(2)})$$

$$A(\bar{B}_s \to \rho^0\gamma_\lambda) = \frac{1}{\sqrt{2}}\lambda_u^{(s)}(E_\lambda + (Q_u - Q_d)PA_{u\lambda}) + \frac{1}{\sqrt{2}}\lambda_c^{(s)}(Q_u - Q_d)PA_{c\lambda} \tag{11}$$

$$A(\bar{B}_s \to \phi^{(8)}\gamma_\lambda) = -\frac{1}{\sqrt{6}}\lambda_u^{(s)}\left(2P_{u\lambda}^{(1)} + 2Q_s P_{u\lambda}^{(2)} - E_\lambda - (Q_u + Q_d - 2Q_s)PA_{u\lambda}\right) \tag{12}$$

$$-\frac{1}{\sqrt{6}}\lambda_c^{(s)}(2P_{c\lambda}^{(1)} + 2Q_s P_{c\lambda}^{(2)} - (Q_u + Q_d - 2Q_s)PA_{c\lambda}) - \sqrt{\frac{2}{3}}\lambda_t^{(s)}(P_{t\lambda} + M_\lambda^{(1)} + Q_s M_\lambda^{(2)}) .$$

Note that the long-distance penguin-type amplitudes with an internal u and c-quark are different in $B^\pm$ and B^0, B_s decays, due to the different electric charge of the spectator quark in the two cases. This is different from the conclusion reached in [9]; present experimental data from CLEO (see Eqs. (56), (57) below) appear to confirm the presence of these long-distance contributions at the 1σ level.

Decays to the SU(3) singlet vector meson $\phi^{(1)} = -\frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$ introduce new amplitudes S_λ, which arise from diagrams similar to PA_λ (see Fig. 1(d)) but with the photon attaching to the quark in the loop or to the spectator in the $\bar{B}$ meson

$$A(\bar{B}^0 \to \phi^{(1)}\gamma_\lambda) = -\frac{1}{\sqrt{3}}\lambda_u^{(d)}(P_{u\lambda}^{(1)} + Q_d P_{u\lambda}^{(2)} + E_\lambda + S_{u\lambda}) \tag{13}$$

$$-\frac{1}{\sqrt{3}}\lambda_c^{(d)}(P_{c\lambda}^{(1)} + Q_d P_{c\lambda}^{(2)} + S_{c\lambda}) - \frac{1}{\sqrt{3}}\lambda_t^{(d)}(P_{t\lambda}^{(1)} + M_\lambda^{(1)} + Q_d M_\lambda^{(2)})$$

$$A(\bar{B}_s \to \phi^{(1)}\gamma_\lambda) = \frac{1}{\sqrt{3}}\lambda_u^{(s)}(P_{u\lambda}^{(1)} + Q_d P_{u\lambda}^{(2)} + E_\lambda + S_{u\lambda}) \tag{14}$$

$$+\frac{1}{\sqrt{3}}\lambda_c^{(s)}(P_{c\lambda}^{(1)} + Q_d P_{c\lambda}^{(2)} + S_{c\lambda}) + \frac{1}{\sqrt{3}}\lambda_t^{(s)}(P_{t\lambda}^{(1)} + M_\lambda^{(1)} + Q_d M_\lambda^{(2)}) .$$

The decay amplitudes into the physical states ϕ, ω are obtained by combining the octet and singlet amplitudes with a mixing angle θ_V as

$$\phi = \phi^{(8)} \cos\theta_V - \phi^{(1)} \sin\theta_V \tag{15}$$

$$\omega = \phi^{(8)} \sin\theta_V + \phi^{(1)} \cos\theta_V \,. \tag{16}$$

The physical value of the mixing angle is close to $\tan\theta_V = \frac{1}{\sqrt{2}}$, corresponding to ideal mixing $\phi = \bar{s}s$, $\omega = -\frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$. Combining the amplitudes for decays into SU(3) eigenstates one finds the following amplitudes for decay into physical states ω, ϕ

$$A(\bar{B}^0 \to \omega\gamma_\lambda) = -\frac{1}{\sqrt{2}}\lambda_u^{(d)}(P_{u\lambda}^{(1)} + Q_d P_{u\lambda}^{(2)} + E_\lambda + \frac{1}{3}PA_{u\lambda} + \frac{2}{3}S_{u\lambda}) \tag{17}$$

$$-\frac{1}{\sqrt{2}}\lambda_c^{(d)}(P_{c\lambda}^{(1)} + Q_d P_{c\lambda}^{(2)} + \frac{1}{3}PA_{c\lambda} + \frac{2}{3}S_{c\lambda}) - \frac{1}{\sqrt{2}}\lambda_t^{(d)}(P_{t\lambda}^{(1)} + M_\lambda^{(1)} + Q_d M_\lambda^{(2)})$$

$$A(\bar{B}^0 \to \phi\gamma_\lambda) = \frac{1}{3}\lambda_u^{(d)}(-PA_{u\lambda} + S_{u\lambda}) + \frac{1}{3}\lambda_c^{(d)}(-PA_{c\lambda} + S_{c\lambda})\,, \tag{18}$$

and

$$A(\bar{B}_s \to \omega\gamma_\lambda) = \frac{1}{\sqrt{2}}\lambda_u^{(s)}(E_\lambda + \frac{1}{3}PA_{u\lambda} + \frac{2}{3}S_{u\lambda}) + \frac{1}{\sqrt{2}}\lambda_c^{(s)}(\frac{1}{3}PA_{c\lambda} + \frac{2}{3}S_{c\lambda}) \tag{19}$$

$$A(\bar{B}_s \to \phi\gamma_\lambda) = -\lambda_u^{(s)}(P_{u\lambda}^{(1)} + Q_s P_{u\lambda}^{(2)} - \frac{1}{3}PA_{u\lambda} + \frac{1}{3}S_{u\lambda}) \tag{20}$$

$$-\lambda_c^{(s)}(P_{c\lambda}^{(1)} + Q_s P_{c\lambda}^{(2)} - \frac{1}{3}PA_{c\lambda} + \frac{1}{3}S_{c\lambda}) - \lambda_t^{(s)}(P_{t\lambda} + M_\lambda^{(1)} + Q_s M_\lambda^{(2)})\,.$$

The radiative decay widths are given, in terms of these amplitudes, by

$$\Gamma(\bar{B} \to V\gamma) = \frac{E_\gamma}{8\pi m_B^2} \sum_{\lambda=L,R} |A(\bar{B} \to V\gamma_\lambda)|^2\,. \tag{21}$$

A. The short-distance amplitude

Due to the smallness of the light quark masses m_d, m_s appearing in the short-distance Hamiltonian (1), relative to the b-quark mass, it couples predominantly to left-handed photons. The amplitude for emitting right-hand polarized photons is suppressed relative to the one for left-handed photons by the quark mass ratio $|P_{tR}/P_{tL}| = \mathcal{O}(m_{d,s}/m_b)$. Therefore, the right-handed amplitude P_{tR} can be neglected to a good approximation.

The left-handed short-distance amplitude can be expressed in terms of the $B \to V$ form factors of the tensor current, defined as

$$\langle V(p', \epsilon)|\bar{q}\sigma_{\mu\nu}b|\bar{B}(p)\rangle = g_+(q^2)\varepsilon_{\mu\nu\lambda\sigma}\epsilon_\lambda^*(p+p')_\sigma + g_-(q^2)\varepsilon_{\mu\nu\lambda\sigma}\epsilon_\lambda^*(p-p')_\sigma \tag{22}$$

$$+h(q^2)\varepsilon_{\mu\nu\lambda\sigma}(p+p')_\lambda(p-p')_\sigma(\epsilon^* \cdot p)\,.$$

Using the relation $\sigma_{\mu\nu}\gamma_5 = \frac{i}{2}\varepsilon_{\mu\nu\alpha\beta}\sigma^{\alpha\beta}$ (corresponding to $\varepsilon^{0123} = 1$), one finds

$$\langle V(p',\epsilon)|\bar q\sigma_{\mu\nu}\gamma_5 b|\bar B(p)\rangle = \\ -ig_+(q^2)[\epsilon^*_\mu(p+p')_\nu-\epsilon^*_\nu(p+p')_\mu]-ig_-(q^2)[\epsilon^*_\mu(p-p')_\nu-\epsilon^*_\nu(p-p')_\mu] \\ -ih(q^2)(\epsilon^*\cdot p)[(p+p')_\mu(p-p')_\nu-(p+p')_\nu(p-p')_\mu]\,. \tag{23}$$

The relevant quantity is the formfactor g_+ at the kinematical point $q^2=0$. This has been computed using light-cone QCD sum rules (LCSR) [20,21] and lattice QCD [22], with results which are in reasonable agreement with each other. The most recent LCSR calculation gave $g_+^{(\rho)}(0)=0.29\pm0.04$ and $g_+^{(K^*)}(0)=0.38\pm0.06$ [21], which compares well with the lattice result $g_+^{(K^*)}(0)=0.32^{+0.04}_{-0.02}$ [22]. A somewhat larger value $g_+^{(K^*)}(0)=0.4$ was recently extracted [23] from the $D\to K^*e\nu$ semileptonic form factors using heavy quark symmetry relations [24].

Using the latter value for $g_+(0)$, one finds for the short-distance penguin amplitudes with an internal t quark

$$P_{tL} = -8\sqrt{2}G_F\frac{e}{16\pi^2}C_7(m_b)m_bm_BE_\gamma g_+(q^2=0) \\ = (1.8\times10^{-6})\left(\frac{g_+(0)}{0.4}\right)\left(\frac{m_b\ (\mathrm{GeV})}{4.2}\right)\ \mathrm{GeV} \tag{24}$$

and $P_{tR}\simeq0$. We used in this numerical estimate the leading-log value for the Wilson coefficient $C_7(m_b)=-0.31$. In the following section we will prove a similar suppression of the right-handed amplitude relative to the left-handed one for the weak annihilation (A) and the W-exchange (E) amplitudes in $\bar B$ decays, to leading order in an expansion in powers of $1/E_\gamma$.

B. Weak annihilation and W-exchange amplitudes

We start by assuming factorization; the corrections to this approximation will be discussed below. The factorized weak annihilation amplitude contributing to the $B^-\to\rho^-\gamma$ decay is written as

$$A_\lambda = \frac{G_F}{\sqrt{2}}a_1\left\{-f_Bp_{B\mu}\langle\rho^-\gamma_\lambda|(\bar du)^\mu_{V-A}|0\rangle+m_{\rho^-}f_{\rho^-}(\epsilon^*_\rho)_\mu\langle\gamma_\lambda|(\bar ub)^\mu_{V-A}|B^-\rangle\right\}\,, \tag{25}$$

corresponding to the photon coupling to the B^- or to the ρ^- constituent quarks respectively. To tree level, the Wilson coefficient a_1 is given by $a_1=C_1(m_b)+C_2(m_b)/N_c=1.02$ [19]. The first term can be computed exactly in the chiral limit with the result

$$p_{B\mu}\langle\rho^-\gamma_\lambda|(\bar du)^\mu_{V-A}|0\rangle = -\langle\rho^-\gamma_\lambda|i\partial_\mu(\bar du)^\mu_{V-A}|0\rangle = -em_{\rho^-}f_{\rho^-}(\epsilon^*_\rho\cdot\epsilon^*_\lambda)+\mathcal{O}(m_u,m_d)\,. \tag{26}$$

We used here the following relations for the divergence of the isovector and isoaxial currents in the presence of an electromagnetic field $\mathcal{A}$ (with $e>0$)

$$i\partial_\mu(\bar q\gamma_\mu\lambda^iq) = -e\mathcal{A}_\lambda\bar q[\hat{\mathcal{Q}},\lambda^i]\gamma_\lambda q-\bar q[\hat m,\lambda^i]q \tag{27}$$

$$i\partial_\mu(\bar q\gamma_\mu\gamma_5\lambda^iq) = -e\mathcal{A}_\lambda\bar q[\hat{\mathcal{Q}},\lambda^i]\gamma_\lambda\gamma_5q-\bar q\{\hat m,\lambda^i\}\gamma_5q\,. \tag{28}$$

$\hat{\mathcal{Q}} = \mathrm{diag}\,(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3})$ and $\hat{m} = \mathrm{diag}\,(m_u, m_d, m_s)$ are the quarks' electric charge and mass matrix, respectively. The result (26) is analogous to a similar one obtained in [25] for the long-distance contribution to the decay $B \to \pi e^+ e^-$.

The remaining contribution can be expressed in terms of the two form factors $f_{V,A}(E_\gamma)$ parametrizing the radiative leptonic decay $B^- \to \gamma e \bar{\nu}$

$$\begin{aligned} &\frac{1}{\sqrt{4\pi\alpha}} \langle \gamma(q, \epsilon_\lambda) | \bar{q} \gamma_\mu (1 - \gamma_5) b | \bar{B}(v) \rangle = \\ &\qquad i\varepsilon(\mu, \epsilon^*_\lambda, v, q) f_V(E_\gamma) + [\epsilon^*_{\lambda\mu}(v \cdot q) - q_\mu(\epsilon^*_\lambda \cdot v)] f_A(E_\gamma) - \frac{1}{E_\gamma}(Q_q - Q_b) f_B m_B (v \cdot \epsilon^*_\lambda) v_\mu \,, \end{aligned} \tag{29}$$

where v denotes the B meson velocity ($p_B = m_B v$). The last term is present only for charged B mesons, and is required for the gauge invariance of the complete decay amplitude, including also the lepton part. Although it is not relevant for real photon processes, it does contribute to decays involving virtual photons, which can probe the $\mu = 0$ component of the electromagnetic current. A detailed derivation of this term is presented in the Appendix.

Combining the two terms in (25) one finds the following result for the weak annihilation helicity amplitudes A_λ

$$A_{L,R} = -\frac{G_F}{\sqrt{2}} a_1 e m_{\rho^-} f_{\rho^-} \left[f_B + E_\gamma (f_A^{(B^-)}(E_\gamma) \mp f_V^{(B^-)}(E_\gamma)) \right] . \tag{30}$$

Eventually the form factors $f_{V,A}(E_\gamma)$ will be extracted from the doubly differential spectrum $d^2\Gamma/dE_e dE_\gamma$ in the radiative leptonic decay $B^\pm \to \gamma e \nu$, which will allow a model-independent calculation of the weak annihilation amplitudes $A_{L,R}$. A considerable simplification can be achieved if the WA amplitudes are expanded in powers of Λ/E_γ using the methods of perturbative QCD for exclusive processes [26]. It has been shown in [27] that the leading terms in the expansion (of order $O(1/E_\gamma)$) of $f_V(E_\gamma)$ and $f_A(E_\gamma)$ are related, and can be expressed in terms of the valence light-cone wave function of the B meson as

$$f_V^{(B^\pm)}(E_\gamma) = \pm f_A^{(B^\pm)}(E_\gamma) = \frac{f_B m_B}{2E_\gamma} \left(Q_u R - \frac{Q_b}{m_b} \right) + \mathcal{O}\left(\frac{\Lambda^2}{E_\gamma^2} \right) . \tag{31}$$

R is a hadronic parameter given by an integral over the B meson light-cone wave function $R = \int_0^\infty dk_+ \psi_B(k_+)/k_+$ (with the normalization $\int_0^\infty dk_+ \psi_B(k_+) = 1$). A similar result is obtained in the quark model, with the identification $R \to 1/m_u$, with $m_u \simeq 350$ MeV the constituent light quark mass [7,9,8]. The relation (31) among $f_{V,A}(E_\gamma)$ implies that a measurement of the photon spectrum in $B^\pm \to \gamma e \nu$ should be sufficient for their extraction (without any knowledge of the B meson light-cone wavefunction).

For the moment, in the absence of such data, the parameter R can be estimated using model wavefunctions, which results in typical values around $R = 2.5 \pm 0.5$ GeV^{-1} [27]. We will use this central value in our estimates below.

It is possible to give a model-independent lower bound for the R parameter, in terms of the B meson binding energy $\bar{\Lambda} = m_B - m_b$. At tree-level this bound reads $R \geq 3/(4\bar{\Lambda})$ [27]. This can be used together with (30) and (31) to derive a lower bound on the magnitude of the A_L amplitude (corresponding to $B^- \to \rho^- \gamma$ decays)

$$A_L \geq \sqrt{2}G_F a_1 e m_{\rho^-} f_{\rho^-}\frac{3Q_u f_B m_B}{8\bar{\Lambda}} = (0.54\times 10^{-6})\left(\frac{a_1}{1.0}\right)\left(\frac{f_B\ (\mathrm{MeV})}{175}\right)\left(\frac{350}{\bar{\Lambda}\ (\mathrm{MeV})}\right)\ \mathrm{GeV}\,. \quad (32)$$

We used in this estimate $f_{\rho^-} = 216$ MeV, as determined from the leptonic decay width of the ρ^0 meson.

To leading order in $1/E_\gamma$, the right-handed amplitude A_R receives contributions only from the first term in (30)

$$A_R = -\frac{G_F}{\sqrt{2}} a_1 e m_{\rho^-} f_{\rho^-} f_B = -(0.07\times 10^{-6})\ \mathrm{GeV}\,, \quad (33)$$

where we used the same values for the hadronic parameters as above. Comparing with (32) one can see that the left/right-handed WA amplitude ratio is enhanced by a factor of more than 8.

The W-exchange amplitude E is given by an expression analogous to (25), with a different phenomenological factorization coefficient $a_2 = C_2(m_b) + C_1(m_b)/N_c = 0.17$ [19]. The result corresponding to the decay $\bar{B}^0 \to \rho^0\gamma$ is written as

$$E_\lambda = G_F a_2 \left\{ -f_B p_{B\mu}\langle \rho^0\gamma_\lambda|(\bar{u}u)^\mu_{V-A}|0\rangle + m_{\rho^0} f_{\rho^0}(\epsilon^*_\rho)_\mu \langle\gamma_\lambda|(\bar{d}b)^\mu_{V-A}|\bar{B}^0\rangle \right\}\,. \quad (34)$$

In the chiral limit only the second term contributes, which gives (with $f_{\rho^0} = \frac{1}{\sqrt{2}} f_{\rho^\pm}$)

$$E_{L,R} = -\frac{G_F}{\sqrt{2}} a_2 e m_{\rho^0} f_{\rho^+} E_\gamma (f_A^{(\bar{B}^0)}(E_\gamma) \mp f_V^{(\bar{B}^0)}(E_\gamma))\,. \quad (35)$$

The form-factors appearing on the RHS are given by an expression analogous to (31) (the sign change is due to our phase convention for the $\bar{B}^0$ state)

$$f_V^{(\bar{B}^0)}(E_\gamma) = -f_A^{(\bar{B}^0)}(E_\gamma) = -\frac{f_B m_B}{2E_\gamma}\left(Q_d R - \frac{Q_b}{m_b}\right) + \mathcal{O}\left(\frac{\Lambda^2}{E_\gamma^2}\right)\,. \quad (36)$$

To the leading order in Λ/E_γ, one finds the lower bound

$$E_L \geq \sqrt{2}G_F a_2 e m_{\rho^0} f_{\rho^+}\frac{3|Q_d| f_B m_B}{8\bar{\Lambda}} = (0.05\times 10^{-6})\left(\frac{a_2}{0.175}\right)\left(\frac{f_B\ (\mathrm{MeV})}{175}\right)\left(\frac{350}{\bar{\Lambda}\ (\mathrm{MeV})}\right)\ \mathrm{GeV}\,. \quad (37)$$

The W-exchange amplitude E is suppressed relative to the WA amplitude A by color-suppression in the ratio a_2/a_1 and by an additional factor $|Q_d/Q_u| = 1/2$. The right-handed E_R amplitude receives contributions only from the higher twist terms in (36) and is therefore suppressed relative to the left-handed amplitude E_R by a factor of $\Lambda/E_\gamma \simeq 0.15$, corresponding to a photon energy in $B \to \rho(K^*)\gamma$ decays $E_\gamma = 2.6$ GeV. This gives the estimate $E_R \simeq 0.15 E_L > (0.007\times 10^{-6})$ GeV.

The remaining amplitudes $P_{u,c}$ are considerably more difficult to evaluate. A great deal of effort has been put into attempts to calculate them, with methods such as simple vector meson dominance [9,13,14], dispersion techniques combined with Regge theory [12] and light-cone QCD sum rules [15–17]. In the following section we will revisit some of these estimates, putting them into perspective relative to the effects computed in this section (see Table I).

Photon helicity	$\lvert P_{t\lambda}\rvert$	$\lvert P_{c\lambda}\rvert$	$\lvert P_{u\lambda}\rvert$	$\lvert A_\lambda\rvert$	$\lvert E_\lambda\rvert$
$\lambda = L$	1.8	0.16	0.03	0.6	0.05
$\lambda = R$	0	0.04	0.007	0.07	0.007

Table I. Estimates of the short-distance and long-distance amplitudes in $B \to \rho\gamma$ decays (in units of 10^{-6} GeV). The estimates of the WA and W-exchange amplitudes A_λ and E_λ used $R = 2.5$ GeV^{-1}.

C. Long-distance amplitudes with internal $c-$ and u-quark loops

The long-distance amplitude $P_{c\lambda}$ induced by the $b \to c\bar{c}q$ part of the weak Hamiltonian (4) is usually assumed to be dominated by the diagram wherein the photon couples to the charm quark loop. In a hadronic language, this contribution arises from the weak decay of the B meson into $V\psi^{(n)}$ (with $\psi^{(n)}$ a $\bar{c}c$ state with the quantum numbers of the photon), followed by the annihilation of the $\psi^{(n)}$ state into a photon [13,14]. For an arbitrary photon virtuality q^2, this long-distance amplitude can be written as a sum over intermediate states

$$A(\bar{B} \to V\psi^{(n)} \to V\gamma_\lambda) = Q_c e \sum_{n,\varepsilon_n} \frac{\langle 0|\bar{c}\gamma \cdot \varepsilon^*_\lambda c|\psi^{(n)}(q,\varepsilon_n)\rangle A(\bar{B} \to V\psi^{(n)})}{q^2 - M_n^2 + iM_n\Gamma_n} . \tag{38}$$

The lowest-lying state contributing to the sum is the J/ψ, with a mass of 3.097 GeV. Therefore, for a real photon, the J/ψ propagator denominator is large, such that its contribution can be expected to be strongly suppressed. These heuristic arguments are supported by an explicit QCD sum rule calculation [17], where the charmed penguin amplitude $P_{c\lambda}$ is found to be less than 5% of the dominant short-distance amplitude P_{tL}. This result is confirmed by existing experimental data, as discussed below.

In the following we will substantiate these qualitative arguments with an estimate of the sum (38), truncating it to a few lowest states. For definiteness we consider the decay $B^- \to \rho^-\gamma$. The weak decay amplitude in (38) can be estimated using factorization, with the result

$$\begin{aligned} A(B^- \to \rho^-\psi^{(n)}) = \frac{G_F}{\sqrt{2}}\lambda_c^{(d)} a_2 f_{\psi^{(n)}} m_{\psi^{(n)}} \Bigg\{ & \frac{2V(m^2_{\psi^{(n)}})}{m_B + m_\rho} i\varepsilon(\epsilon^*_\psi, p_B, p_\rho, \epsilon^*_\rho) \\ & -(m_B + m_\rho)A_1(m^2_{\psi^{(n)}})(\epsilon^*_\psi \cdot \epsilon^*_\rho) + \frac{2A_2(m^2_{\psi^{(n)}})}{m_B + m_\rho}(\epsilon^*_\rho \cdot q)(\epsilon^*_\psi \cdot p_B) \Bigg\} \end{aligned} \tag{39}$$

The form-factors appearing here are defined in the usual way (with $q = p' - p$)

$$\langle \rho^-(p',\epsilon)|\bar{u}\gamma_\mu b|B^-(p)\rangle = \frac{2V(q^2)}{m_B + m_\rho} i\varepsilon(\mu, p, p', \epsilon^*) \tag{40}$$

$$\begin{aligned} \langle \rho^-(p',\epsilon)|\bar{u}\gamma_\mu\gamma_5 b|B^-(p)\rangle = {} & 2m_\rho A_0(q^2)\frac{\epsilon^* \cdot q}{q^2} q_\mu + (m_B + m_\rho)A_1(q^2)\left(\epsilon^*_\mu - \frac{\epsilon^* \cdot q}{q^2} q_\mu\right) \\ & -A_2(q^2)\frac{\epsilon^* \cdot q}{m_B + m_\rho}\left(p_\mu + p'_\mu - \frac{m_B^2 - m_\rho^2}{q^2} q_\mu\right) , \end{aligned} \tag{41}$$

and the decay constant of the ψ state is given by $\langle 0|\bar{c}\gamma_\mu c|\psi(q,\epsilon)\rangle = m_\psi f_\psi \epsilon_\mu$. Inserting (39) into the sum (38) and performing the sum over the $\psi^{(n)}$ polarization one finds the following result for the long-distance charmed penguin contribution to the $B^- \to \rho^-\gamma$ amplitude

$$\begin{aligned} P_{c\lambda}(B^- \to \rho^-\gamma_\lambda) &= Q_c e \frac{G_F}{\sqrt{2}} a_2 \frac{2m_B}{m_B+m_\rho} \sum_n f^2_{\psi^{(n)}} \\ &\times \left\{ V(0)\, i\varepsilon(q,\epsilon^*_\lambda,\epsilon^*_\rho,v) - A_2(0)[(v\cdot q)(\epsilon^*_\rho\cdot\epsilon^*_\lambda) - (v\cdot\epsilon^*_\lambda)(q\cdot\epsilon^*_\rho)]\right\} . \end{aligned} \tag{42}$$

To put the result into this (explicitly gauge-invariant) form, one must assume the following relation between the form factors A_1 and A_2 at $q^2 = 0$ [14]

$$A_1(0) = A_2(0)\frac{m_B - m_\rho}{m_B + m_\rho} . \tag{43}$$

While the approximate numerical values of these form factors $V^{B\rho}(0) = 0.34, A_1^{B\rho}(0) = 0.26, A_2^{B\rho}(0) = 0.22$ [21] do not satisfy exactly the relation (43), it is worth noting that it does hold true to leading order in a large energy expansion in powers of $1/E_\rho$. In [30], the form-factors for the decay of a B meson into a light pseudoscalar (vector) meson have been analyzed using an effective theory for the final state quarks [31]. To leading twist, all these form factors can be expressed in terms of two universal functions $\zeta_\perp(E_\rho)$ and $\zeta_\parallel(E_\rho)$

$$V(q^2) = \left(1 + \frac{m_\rho}{m_B}\right)\zeta_\perp(E_\rho) \tag{44}$$

$$A_1(q^2) = \frac{2E_\rho}{m_B + m_\rho}\zeta_\perp(E_\rho) \tag{45}$$

$$A_2(q^2) = \left(1 + \frac{m_\rho}{m_B}\right)\left[\zeta_\perp(E_\rho) - \frac{m_\rho}{E_\rho}\zeta_\parallel(E_\rho)\right] . \tag{46}$$

The leading twist form factors satisfy certain relations, which are very similar (but not completely identical) to relations previously derived in the quark model [32]. This is not completely unexpected, considering that to leading twist only the meson's valence degrees of freedom are relevant, which esentially coincides with the quark model picture.

Using the relations (45), (46) one can see that the condition (43) is indeed satisfied, up to corrections of higher order in m_ρ/E_ρ, proportional to $\zeta_\parallel$. The relations (44), (46) imply also that only left-handed photons are emitted to leading twist. The relevant helicity amplitudes are given by

$$P_{cL,R} = Q_c e \frac{G_F}{\sqrt{2}} a_2 \frac{2m_B}{m_B + m_\rho} E_\gamma (V(0) \pm A_2(0)) \sum_n f^2_{\psi^{(n)}} . \tag{47}$$

Using the realistic values for form-factors quoted above, one finds that the left/right-handed helicity amplitudes ratio is enhanced by about a factor of 5. Absolute values for the $|P_{c\lambda}|$ helicity amplitudes obtained by keeping only the lowest two states in (38) are quoted in Table I. The corresponding decay constants can be extracted from their leptonic widths and are given by $f_{\psi^{(1)}} = 395$ MeV, $f_{\psi^{(2)}} = 293$ MeV [14].

A similar analysis can be performed for the long-distance amplitude $P_{u\lambda}$ describing quark diagrams with u-quark loops, which is given by a sum analogous to (38)

$$A(\bar{B} \to VV' \to V\gamma_\lambda) = e \sum_{V'=\rho^0,\omega,\dots} \frac{\langle 0|j_{\rm e.m.} \cdot \varepsilon^*_\lambda|V'(q,\varepsilon)\rangle A_C(\bar{B} \to VV')}{q^2 - M^2_{V'} + iM_{V'}\Gamma_{V'}} \,. \tag{48}$$

The weak amplitudes $A(\bar{B} \to VV')$ receive, in general, both color-allowed and color-suppressed contributions (in contrast to the decays $B \to V\psi$ which are purely color-suppressed). For example, in $B^- \to \rho^-\gamma$ decay, the color-allowed component gives a WA-type contribution to the radiative decay. To avoid double counting of amplitudes, we will keep therefore only the color-suppressed component of the weak amplitude in (48) (symbolized by the subscript C). Our analysis is different in this respect from other VMD estimates of this amplitude [14].

Keeping only the contributions of the lowest two states $V' = \rho^0, \omega$, one finds

$$P_{uL,R} = \frac{G_F}{\sqrt{2}} e a_2 f_{\rho_0} m_{\rho_0} \frac{2m_B}{m_B + m_\rho} E_\gamma \left(\frac{f_{\rho_0}}{m_\rho} + \frac{f_\omega}{m_\omega}\right) (V(0) \pm A_2(0)) \,. \tag{49}$$

In writing this result we used the nonet symmetry relation $\langle 0|\bar{u}\gamma_\mu u|\rho^0\rangle = \langle 0|\bar{u}\gamma_\mu u|\omega\rangle$, valid in the large N_c limit or the quark model. The corresponding numerical results are shown in Table I.

The enhancement of the left-handed photon amplitudes relative to the right-handed ones in the long-distance contributions we found is somewhat surprising. Calculations of these amplitudes using different methods such as QCD sum rules [15–17] and perturbative QCD [6] find a similar result. Such an enhancement does not necessarily hold in the presence of new physics. In certain extensions of the Standard Model, such as the left-right symmetric model, the right-handed helicity amplitude can become large (proportional to the virtual heavy fermion mass) [33]. Such effects lead to observable consequences such as significant mixing-induced CP asymmetries in weak radiative $B \to V\gamma$ decays [34].

To summarize the results of this section, the rare radiative decays are dominated by the amplitudes corresponding to left-handed photons. The dominant long-distance contribution to the $B^- \to \rho^-\gamma$ decay comes from the WA mechanism; the corresponding amplitude A_λ is calculable in a model-independent way from experimental data. The left-handed amplitudes satisfy an approximate hierarchy of sizes

$$E \simeq P_u \simeq 0.4 P_c \,, \quad P_c \simeq 0.2 A \,, \quad A \simeq 0.3 P_t \,. \tag{50}$$

The relative magnitude of the components with different CKM coefficients can be obtained by combining these estimates for the amplitudes with the corresponding CKM factors.

D. Nonfactorizable corrections

The leading corrections to the factorization result (25) for the WA amplitudes come, in perturbation theory, from the graphs in Fig. 2, with one gluon exchanged between the B and ρ quarks.

Let us take for definiteness the final vector meson to be moving along the positive z axis. Momentum conservation $m_B v = q + p$ is expressed in terms of light-cone components as $p_+ = m_B$ and $m_B = 2E_\gamma + p_-$ (we define the light-cone coordinates as $p_\pm = p^0 \pm p^3$). Then to leading order $O(1)$ in an expansion in powers of $\Lambda/p_+ = \Lambda/m_B$, the ρ meson can be

represented by its valence component $|q(p_1)\bar{q}(p_2)\rangle$ with parallel momenta $p_1 = (yp_+, 0_-, 0_\perp)$ and $p_2 = ((1-y)p_+, 0_-, 0_\perp)$. In a tensor language this corresponds to the ρ wave function

$$(\Psi_\rho)_{\alpha\beta} = \frac{\sqrt{N_c}}{8\sqrt{2}} p_+ (\gamma_- \not\varepsilon)_{\alpha\beta} \phi_\perp(y) \tag{51}$$

where $\phi_\perp(y)$ is the twist-2 chiral-odd structure function as appropriate for a transversely polarized ρ meson. It satisfies the normalization condition

$$\int_0^1 dy \phi_\perp(y) = \sqrt{\frac{2}{N_c}} f_\rho^T\,, \tag{52}$$

where the decay constant $f_\rho^T = (160 \pm 10)$ MeV [28] is defined as

$$\langle 0|\bar{u}\sigma_{\mu\nu} d|\rho^-(p,\varepsilon)\rangle = i f_\rho^T (\varepsilon_\mu p_\nu - \varepsilon_\nu p_\mu)\,. \tag{53}$$

The B meson is represented by the tensor wavefunction [27]

$$(\Psi_B)_{\alpha\beta} = \frac{\sqrt{N_c}}{\sqrt{2}k_+} \psi_B(k_+, \vec{k}_\perp) \left\{ (k_+ + \vec{\alpha}_\perp \cdot \vec{k}_\perp)\Lambda_+ \frac{1+\not v}{2}\gamma_5 \right\}_{\alpha\beta}\,, \quad \Lambda_+ = \frac{\gamma_-\gamma_+}{4}\,. \tag{54}$$

In contrast to the light mesons π or ρ, the leading twist description of a heavy meson involves in general (beyond tree level) also the transverse motion of the heavy quark.

Each of the four diagrams in Fig. 2a-d contains IR divergences; however, an explicit calculation shows that they cancel in the sum. Therefore, (at least to one-loop order) the leading-twist nonfactorizable contribution is computable in perturbation theory as a convolution of the B and ρ light-cone wave functions with a IR-finite hard scattering amplitude T_H

$$\delta_{n.f.} A_L = \int_0^\infty dk_+ \int_0^1 dy \phi_\perp(y,\mu)\psi_B(k_+,\mu) T_H(k_+, y, \mu) + \text{higher twist}\,. \tag{55}$$

(This is analogous to a result recently found in [35] for nonleptonic decays $B \to \pi\pi$.) However, it is easy to see that the hard-scattering amplitude vanishes in the limit of massless final quarks $T_H = O(m_{u,d})$. Technically, in this limit, the ρ side of the matrix element contains only traces of an odd number of γ matrices, which vanish.

None of these conclusions depends on the fact that the photon is on-shell $q^2 = 0$, such that a similar result can be derived for $B \to \rho_\perp e^+ e^-$ decays, mediated by a virtual photon. However, a nontrivial nonfactorizable correction is obtained for a longitudinally polarized ρ or a pion in the final state.

In conclusion, the one-loop nonfactorizable correction vanishes identically in the chiral limit and to leading-twist order. Therefore the factorized result (25) can be expected to give a very good approximation to the weak annihilation amplitude A.

III. APPLICATIONS

A. Implications from experimental data

The CLEO Collaboration [4] recently reported the branching ratios for the $B \to K^* \gamma$ exclusive modes

$$\mathcal{B}(B^\pm \to K^{*\pm}\gamma) = (3.76^{+0.89}_{-0.83} \pm 0.28) \times 10^{-5} \tag{56}$$
$$\mathcal{B}(B^0 \to K^{*0}\gamma) = (4.55^{+0.72}_{-0.68} \pm 0.34) \times 10^{-5}\,. \tag{57}$$

These results give the following ratio for the charge-averaged radiative decay widths

$$\frac{\Gamma(B^0 \to K^{*0}\gamma)}{\Gamma(B^\pm \to K^{*\pm}\gamma)} = \frac{\tau_{B^\pm}}{\tau_{B^0}} \frac{\mathcal{B}(B^0 \to K^{*0}\gamma)}{\mathcal{B}(B^\pm \to K^{*\pm}\gamma)} = 1.29 \pm 0.55\,. \tag{58}$$

We used here the lifetime ratio of charged and neutral B mesons $\tau(B^\pm)/\tau(B^0) = 1.066 \pm 0.024$ [36].

Isospin symmetry constrains the short-distance amplitude P_{tL} to be the same in both decays (56), (57). Therefore the deviation of the ratio (58) from 1 can only come from the long-distance contribution $M^{(2)} - P_c^{(2)}$ arising from the photon coupling to the spectator quark. Using the unitarity of the CKM matrix one can rewrite $\lambda_c^{(s)} = -\lambda_u^{(s)} - \lambda_t^{(s)}$ in (6) and (8). Furthermore, neglecting the small CKM factor $\lambda_u^{(s)}$, these amplitudes can be written to a good approximation as

$$A(B^- \to K^{*-}\gamma) = \lambda_t^{(s)} \left(P_t + (M^{(1)} - P_c^{(1)}) + \frac{2}{3}(M^{(2)} - P_c^{(2)}) \right) \tag{59}$$
$$A(\bar{B}^0 \to \bar{K}^{*0}\gamma) = \lambda_t^{(s)} \left(P_t + (M^{(1)} - P_c^{(1)}) - \frac{1}{3}(M^{(2)} - P_c^{(2)}) \right)\,. \tag{60}$$

The short-distance amplitude P_{tL} alone would give the following branching ratios

$$\mathcal{B}_{\text{s.d.}}(B^\pm \to K^{*\pm}\gamma) = 4.6 \times 10^{-5}\,, \qquad \mathcal{B}_{\text{s.d.}}(B^0 \to K^{*0}\gamma) = 4.5 \times 10^{-5}\,, \tag{61}$$

where we used the estimate for P_{tL} shown in Table I together with $\lambda_t^{(s)} = 0.039$. The B meson lifetimes were taken as $\tau_{B^\pm} = 1.64 \pm 0.02$ ps, $\tau_{B^0} = 1.55 \pm 0.03$ ps [36]. Assuming that the central values of the experimental numbers (56), (57) will be confirmed by future, more precise determinations, one is led to conclude therefore that the long-distance effects interfere among themselves destructively in B^0 decays, whereas in $B^\pm$ they could be significant, and contribute up to 10% of the total amplitude. This agrees with the theoretical estimates of [6] and disfavor a non-SM explanation of the isospin breaking in the ratio (58) based on an enhanced gluonic penguin as proposed in [37].

In the SU(3) limit the long-distance contributions to B^0 and B_s decays are the same (see (7)-(12)). Therefore one can use the observed $B^0 \to K^{*0}\gamma$ branching ratio to make predictions for other strangeness-changing radiative decays of these mesons. Neglecting the OZI-suppressed amplitudes $S_{c\lambda}$ and $PA_{c\lambda}$, one obtains the following prediction (together with $\tau_{B_s} = 1.46 \pm 0.06$ ps [36])

$$\mathcal{B}(B_s \to \phi\gamma) = \frac{\tau_{B^0}}{\tau_{B_s}} \mathcal{B}(B^0 \to K^{*0}\gamma) \simeq 4.8 \times 10^{-5}\,. \tag{62}$$

Using the estimate for E_L in Table I, and neglecting again the contributions of the OZI-suppressed annihilation penguin PA and SU(3) singlet S amplitudes, one can predict very small rates for the B_s modes (we used here $|V_{ub}| = 1.6 \times 10^{-2}$)

$$\mathcal{B}(B_s \to \rho^0\gamma) \simeq \mathcal{B}(B_s \to \omega\gamma) \simeq 0.3 \times 10^{-8} \,. \tag{63}$$

B. Determining $|V_{td}|$

The main interest in observing the exclusive charmless radiative decay $B^\pm \to \rho^\pm\gamma$ is connected with the possibility of determining the CKM matrix element $|V_{td}|$. The amplitude for this decay is related by U-spin symmetry to that for the $b \to s\gamma$ mode $B^\pm \to K^{*\pm}\gamma$

$$A(B^- \to \rho^-\gamma_L) = \lambda_u^{(d)} a_L + \lambda_t^{(d)} p_L \,, \qquad A(B^- \to K^{*-}\gamma_L) = \lambda_u^{(s)} a'_L + \lambda_t^{(s)} p'_L \,. \tag{64}$$

In the SU(3) symmetry limit $a_L = a'_L = A_L - P_{cL}, p_L = p'_L = P_{tL} + M_L - P_{cL}$. In the absence of the a, a' terms, the ratio of the two amplitudes is equal to $|\lambda_t^{(d)}/\lambda_t^{(s)}| = |V_{td}/V_{ts}|$ (up to SU(3) breaking corrections), which offers a way for determining $|V_{td}|$.

In the general case, however, the $(\rho^-\gamma_L)$ amplitude can be written as

$$A(B^- \to \rho^-\gamma_L) = \lambda_t^{(d)} p(1 - \frac{|\lambda_u^{(d)}|}{|\lambda_t^{(d)}|} e^{i\alpha}\varepsilon_A e^{i\phi_A}) \,, \tag{65}$$

with $\varepsilon_A e^{i\phi_A} \equiv a/p \simeq A_L/P_{tL}$ (where, following the estimates of Sec. II we neglected the charmed penguin P_{cL} and gluonic penguin M_L amplitudes relative to the short-distance amplitude P_{tL} and the WA amplitude A_L). The estimates of the preceding section give $\varepsilon_A \simeq 0.3$, and $\phi_A = 0$ to leading twist. Furthermore, global analyses of the unitarity triangle suggest that the ratio of CKM factors appearing in this amplitude is subunitary [38]

$$\frac{|\lambda_u^{(d)}|}{|\lambda_t^{(d)}|} \simeq \left|\frac{V_{ub}}{V_{cb}}\right| \cdot \left|\frac{V_{ts}}{V_{td}}\right| \simeq 0.4 \,. \tag{66}$$

The estimates of Sec. II.B show that the total rate for B^- (B^+) radiative decay is dominated by left-hand (right-hand) polarized photons. Neglecting the small right-handed (left-handed) component, which gives only a second order contribution to the ratio, one finds for the ratio of the charge-averaged rates

$$\begin{aligned} &\frac{\mathcal{B}(B^- \to \rho^-\gamma) + \mathcal{B}(B^+ \to \rho^+\gamma)}{\mathcal{B}(B^- \to K^{*-}\gamma) + \mathcal{B}(B^+ \to K^{*+}\gamma)} \\ &= \left(\frac{|p|}{|p'|} \cdot \frac{|V_{td}|}{|V_{ts}|}\right)^2 \left\{1 - \frac{|\lambda_u^{(d)}|}{|\lambda_t^{(d)}|}\varepsilon_A \cos\alpha\cos\phi_A + \left(\frac{|\lambda_u^{(d)}|}{|\lambda_t^{(d)}|}\varepsilon_A\right)^2\right\} \,. \end{aligned} \tag{67}$$

The factor in the braces is bounded from above and below by using the inequality

$$1 + x + x^2 \le 1 - x\cos\alpha\cos\phi_A + x^2 \le 1 - x + x^2 \,, \quad x \equiv \frac{|\lambda_u^{(d)}|}{|\lambda_t^{(d)}|}\varepsilon_A \simeq 0.12 \,. \tag{68}$$

Assuming, conservatively, $x = 0.2$ (which is on the upper side of the estimates for this parameter) gives an uncertainty of about 20% in the determination of $|V_{td}|$ arising from the factor in braces in (67). The SU(3) breaking factor in the ratio of the penguin amplitudes $|p|/|p'| \simeq g_+^{(\rho)}(0)/g_+^{(K^*)}(0) = 0.76 \pm 0.22$ is known with a uncertainty of about 30% (we used here the results of the LCSR calculation [21]). Future lattice QCD calculations will hopefully improve the precision with which this ratio is known. Adding these errors in quadrature one finds a theoretical error in the determination of $|V_{td}|$ from the ratio (67) of about 35%. The leading twist result $\phi_A = 0$ implies that the CP asymmetry in $B^\pm \to \rho^\pm\gamma$ can be expected to be very small ($A_{CP} \propto \sin\phi_A$).

The above estimate for the long-distance contribution ε_A is in general agreement with the quark model of [9,14] and QCD sum rules [15,16] calculations. The advantage of our approach will become apparent once experimental data on radiative leptonic B decays become available. Such data could be used, as discussed in Sec. II, to actually determine the WA amplitude A_L, and thereby the long-distance contamination ε_A.

IV. CONCLUSIONS

Present experimental data from CLEO on exclusive radiative weak decays [4] are sufficiently precise such that the long-distance effects are beginning to be observable. A good control over the magnitude of these effects is important for an assessment of the uncertainty they induce into the extraction of $|V_{td}|$ by combining $B^- \to \rho^-\gamma$ and $B^- \to K^{*-}\gamma$ decays [9,14]. Unfortunately, these effects have proved notoriously difficult to treat in any systematic way.

In the present paper we focus on a certain class of long-distance contributions, arising from weak annihilation and W-exchange quark diagram topologies. We argue that the former gives the dominant long-distance correction to the amplitude for $B^- \to \rho^-\gamma$ decays. These corrections can be computed reliably using factorization, in terms of form-factors observable in radiative leptonic decays $B^+ \to \gamma e^+\nu$. The nonfactorizable corrections are shown to be very small, as they appear only at nonleading twist. A similar method can be used to compute the W-exchange-type long-distance contribution, relevant for the weak radiative B_d and B_s decays.

Furthermore, to the leading order of an expansion in powers of Λ/E_γ, one can show that the coupling of left-handed photons in the long-distance amplitude is greatly enhanced relative to that of right-handed photons, just as in the short-distance part of the amplitude. Such an enhancement had been previously observed in QCD sum rule calculations of the long-distance amplitude [15]; our approach clarifies the theoretical limit in which this enhancement holds and quantifies the magnitude of the corrections to it.

Our results should allow one to reduce the model-dependence of the leading long-distance effects in $B^- \to \rho^-\gamma$ decays, and achieve a better control over the theoretical error in the corresponding determination of $|V_{td}|$.

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APPENDIX A: WARD IDENTITIES FOR LONG-DISTANCE MATRIX ELEMENTS

We present in this Appendix a set of constraints on certain long-distance contributions to weak radiative decays of B mesons. These constraints follow from a Ward identity expressing the conservation of the electromagnetic current. Let us consider the following matrix element of a local operator $\mathcal{O}$, to first order in electromagnetism and to all orders in the strong coupling

$$\langle\gamma(q,\epsilon)f|\mathcal{O}(0)|B(v)\rangle = -ie\epsilon^*_\mu\int \mathrm{d}^4x e^{iq\cdot x}\langle f|\mathrm{T}j^{\mathrm{e.m.}}_\mu(x)\,\mathcal{O}(0)|B(v)\rangle\,, \tag{A1}$$

for any hadronic final state f. The operator $\mathcal{O}$ can be any quark bilinear including gluon fields, or even a four-quark operator. The electromagnetic current includes contributions from both the light and heavy quarks $j^{\mathrm{e.m.}}_\mu = \bar q\hat{\mathcal{Q}}\gamma_\mu q + \frac{2}{3}\bar c\gamma_\mu c - \frac{1}{3}\bar b\gamma_\mu b$. A relation similar to (A1) can be written for the matrix element with the photon replaced with a dilepton pair in the final state, coupling through a virtual photon to the hadronic system.

The conservation of the electromagnetic current implies, in the standard way, a Ward identity for the matrix element of the time-ordered product in (A1)

$$-iq_\mu\int \mathrm{d}^4x e^{iq\cdot x}\langle f|\mathrm{T}j^{\mathrm{e.m.}}_\mu(x)\,\mathcal{O}(0)|B(v)\rangle = \int \mathrm{d}^3x e^{-i\vec q\cdot\vec x}\langle f|[j^{\mathrm{e.m.}}_0(\vec x)\,,\mathcal{O}(\vec 0)]|B(v)\rangle\,. \tag{A2}$$

The commutator on the RHS is nonvanishing only if the operator $\mathcal{O}$ carries an electric charge, as in the case of decays induced by the weak charged current such as $B^+\to\gamma e^+\nu$, $B\to\pi(\rho)\gamma e^+\nu$ or $\bar B\to D^{(*)}\gamma e^+\nu$. In the following we analyze the simplest such case, the radiative leptonic decays of a B meson, for which $\mathcal{O} = \bar b\gamma_\nu\gamma_5 q$ and $|f\rangle = |0\rangle$.

The equal-time commutator on the RHS of (A2) can be computed explicitly, with the result

$$\begin{aligned}-iq_\mu\int \mathrm{d}^4x e^{iq\cdot x}\langle 0|\mathrm{T}j^{\mathrm{e.m.}}_\mu(x)\,(\bar b\gamma_\nu\gamma_5 q)(0)|B(v)\rangle &= (Q_b - Q_q)\langle 0|\bar b\gamma_\nu\gamma_5 q|B(v)\rangle \\ &= (Q_b-Q_q)f_B m_B v_\mu\,.\end{aligned} \tag{A3}$$

The most general parametrization of the matrix element on the LHS can be written in terms of five form-factors $f_i(q^2, v\cdot q)$

$$-i\int \mathrm{d}^4x e^{iq\cdot x}\langle 0|\mathrm{T}j^{\mathrm{e.m.}}_\mu(x)\,(\bar b\gamma_\nu\gamma_5 q)(0)|B(v)\rangle = f_1 g_{\mu\nu} + f_2 v_\mu v_\nu + f_3 q_\mu q_\nu + f_4 q_\mu v_\nu + f_5 v_\mu q_\nu\,. \tag{A4}$$

The Ward identity (A3) implies two constraints on these form-factors

$$(v\cdot q)f_2 + q^2 f_4 = (Q_b-Q_q)f_B m_B\,,\qquad f_1 + q^2 f_3 + (v\cdot q)f_5 = 0\,. \tag{A5}$$

For the case of a real photon $q^2=0$, these constraints fix uniquely the form-factor $f_2(0, v\cdot q)$, and relate $f_1(0,v\cdot q)$ and $f_5(0, v\cdot q)$. From Eq. (A1) one finds thus the following result for the matrix element of the axial weak current

$$\langle\gamma(q,\epsilon)f|\bar b\gamma_\mu\gamma_5 q|B(v)\rangle = -f_5[(v\cdot q)\epsilon^*_\mu - (v\cdot\epsilon^*)q_\mu] + (v\cdot\epsilon^*)v_\mu\frac{1}{v\cdot q}(Q_b-Q_q)f_B m_B\,, \tag{A6}$$

which is the same as the result presented in text Eq. (29), with the identification $f_5 = f_A$.

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FIGURES

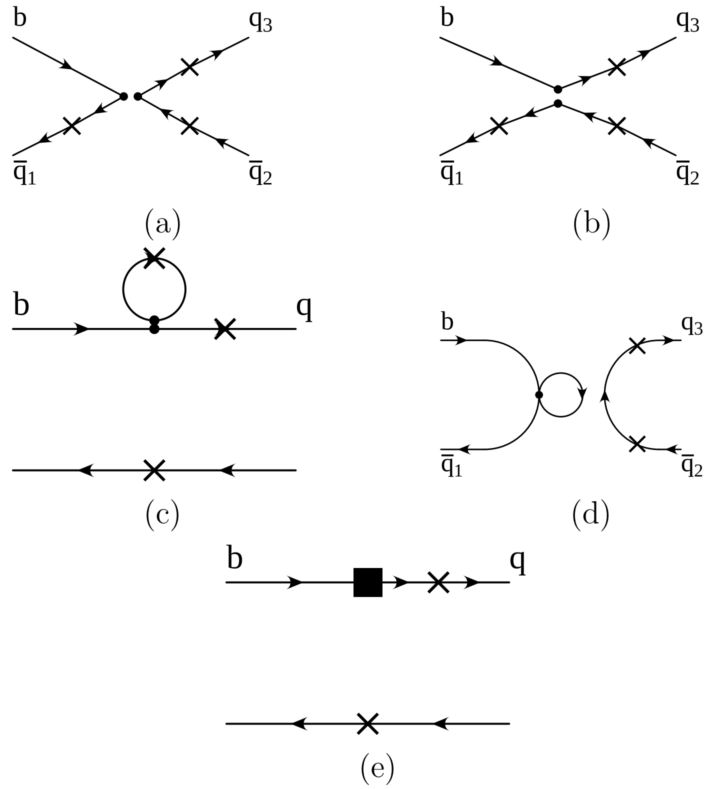

FIG. 1. Quark diagrams contributing to $\bar{B} \to V\gamma$ decays. The cross marks the attachment of the photon line. a) weak annihilation amplitudes $A^{(i)}$, with $i = 1, 2, 3$ corresponding to the photon attaching to each of the three quark lines $\bar{q}_1, \bar{q}_2, q_3$; b) W-exchange amplitudes $E^{(i)}$, $i = 1, 2, 3$; c) penguin amplitudes $P^{(1)}_{q'}$ (the photon is attached to the $\bar{q} = \bar{d}, \bar{s}$ quark or the quark $q' = u, c$ running in the loop) and $P^{(2)}_{q'}$ with the photon attached to the spectator quark; d) annihilation penguin amplitudes $PA_{q'}$; e) amplitudes with one insertion of the gluonic penguin $M^{(1)}$ (photon attaching to the $\bar{q}$ line) and $M^{(2)}$ (photon attaches to the spectator quark). The box denotes one insertion of the operator Q_8.

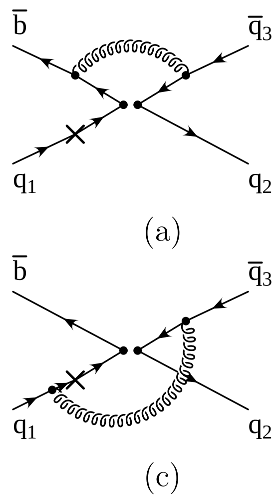

FIG. 2. Corrections to factorization in $B \to V\gamma$ decays. Only those diagrams are shown which contain IR divergences. The total IR divergence cancels in their sum. The cross denotes the attachment of the photon line.